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90年代太阳模型和太阳震荡研究进展(II)：太阳振荡研究进展

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摘 要

太阳振荡研究现已成为研究太阳内部性质的新手段，也成为检验太阳模型构造时输入物理参量的最重要工具。90年代以来理论与观测日震频率的差别已随输入物理参量及太阳振荡理论的改进而大为减小，可是现有的差别仍远大于观测误差。由日震反演可对太阳内部对流区、表面氦丰度及自转随纬度和径向的分布都有更多了解。太阳振荡的湍动随机激发及激发源的位置都已得到研究，不过现在问题还未完全解决。今后一方面要探测更多的振动方式，另一方面也需要解决不同观测者得到的结果存在系统差的问题，而最外层的非绝热现象及理论与观测存在差别仍是最关键的难题。

关 键 词 太阳：振荡 — 太阳：基本参量 — 太阳：内部

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Research on the Solar Model and Oscillation in 1990s (II): Progress in Research on the Solar Oscillations

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Abstract

Helioseismology has become a new tool for the study of the solar interior as well as one of

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the most important devices to determine the quality of the the input physics that enters the solar models. The discrepancy between theoretical and observational frequencies has drastically decreased in the 1990s due to modified input physics and improved stellar oscillation theory. However, the discrepancy is still bigger than the observational error. From frequency inversion studies we know more about the solar convection zone, the surface helium abundance and rotational velocity distribution as a function of depth and latitude. The stochastic excitation mechanism by turbulence and the location of the excitation sources are well studied, but not completely understood. More work is needed for the detection of new oscillation modes and also to explain the systematic discrepancy between different observers. The non-adiabatic physics in the outermost layer and the difference between theoretical and observational frequencies especially require closer attention.

Key words Sun: oscillations—Sun: fundamental parameters—Sun: interior

1 Introduction

As the nearest star from us, the Sun has been carefully observed for thousands of years. But since we cannot see inside, we can only directly observe the surface part outside the solar convection zone. When we want to know something more about its interior, we construct a theoretical model of the Sun based on some assumptions, and then fit the model with the surface characteristics we observed. Once they coincide with each other, we then use the model to study its inner structure. The problem is that the information we can get from solar surface features is limited, while there are too many possible factors that may influence solar structure, so that sometimes even some different physical input can yield models with still similar surface features. Thus the results from model construction are sometimes questionable.

The discovery of solar surface 5-minutes oscillations has enlightened our study of solar interior models. At present time, based on those space satellites and global observational networks, we have found as many as 10^7 different solar p-modes, with uncertainty of only 10^{-5}. These oscillation frequencies give us much more constraints on our models than before. In order to fit all these different observational modes, our theoretical models must have much higher accuracy.

We can use those observational frequencies directly to investigate the propagation of sound wave in the solar interior, and then inverse a cavity which can allow these modes to survive as standing waves. The difference of this cavity and our theoretical model may lead us find out those areas that need to be improved, and, thus, let us to know the physical processes that have been missed or misused. These results can even be used for the study of other celestial objects far from us, so that we can have more knowledge on more astronomical phenomenon. The study of exciting and damping mechanisms of solar oscillation can also tell us more about the interaction among convective motion, radiative field and oscillation process, as well as the variation of solar interior and magnetic field on helioseismology. Such a connection between the study of solar active region and global structure has let present day solar physics study a wider field than before.

The study of helioseismology gives us more constraints on model construction, and forces us to study more precisely on the input parameters, such as equation of state, opacity, nuclear reaction rates, etc. And the Sun has even become one of the most important laboratories on distinguishing different treatment on these input physics, whose differences have been sheltered by some other factors or errors in ground laboratories. The discrepancy of solar surface helium abundance between inversion and modeling has also reminded us the ignored helium and heavy elements diffusion process. The frequencies are one of the most important test stones in distinguishing those models constructed to analyse solar surface lithium depletion, and the study of low degree solar p-modes reduces the possibility of analysing solar neutrino puzzle in astrophysics.

In the 1990s, helioseismology, albeit still young, has become one of the most vigorous research areas. It is now a very important tool to study the solar interior. From satellites and ground based observation networks, more and more high accuracy and high resolution data will become available, and more phenomena to be analysed will appear. In this paper we will focus on the progress in the study of solar oscillations in 1990s, as well as on some problems that still exist, so as to give researchers who work in related field an overview. In section 2 we will introduce the new discoveries from observation. In section 3 more achievements and problems in theoretical studies will be reviewed. Discussion and conclusion will be in section 4.

2 Progress in Observation

Solar oscillation is composed of a series of eigenfunction standing wave modes which travel through the global Sun. Because of the oscillation, either solar surface radial velocity field, intensity of the continuous spectrum, and the spectra lines will be affected. Usually what we observed for solar oscillation is to measure the variation of these values with time and space. One of the most popular ways is to study the Doppler effect of the radial velocity caused by oscillation by comparing the red with the blue wing intensity of the observing spectra lines. The continuous spectrum intensity variation is more useful for low degree modes, while the subtraction intensity of spectra line center and its nearby continuous spectrum can get more accurate results for high degree modes than the measurement of the Doppler effect, because this value will get less influence from the Earth atmosphere turbulence [207].

As the maximum velocity amplitude for solar p-mode is as small as $20\text{cm} \cdot \text{s}^{-1}$, no obvious information can be get from only several periods of observation, unlike to what we are able to do with usual variable stars. Only after the correlation of thousands of periods, we can obtain those high resolution results. Fortunately, the solar p-mode are all concentrated to the frequency of $\nu = 2 - 6\text{mHz}$, with its oscillation power spectrum at the period of around 5 minutes, so that we can get thousands of circles in several months' continuous observation. As the oscillation itself is correlated, when N different period of data are added together, the signal will enhance for N times, while the noise, which is not correlated, will increase by only $\sqrt{N}$ times. Hence, we can get pretty high signal-to-noise ratio results. By now the modes we can detect are more than 10^7, with uncertainty of about 10^{-5} [208].

With several decades of observational data, we have got those $0 \leq l \leq 1860$ modes, with frequency of $1.0 \leq \nu \leq 5.3$mHz p-modes. When compared with theoretical values, the discrepancy is from about 2μHz for low degree modes to about 30μHz for high degree modes[209–213]. Based on SOHO satellite [214] and ground networks [215,216], more accurate new modes will be available, which are highly desired by theorists [143]. Problem is that, because what we observe is only one part but not the whole solar surface, some of the energy will leak out in the space spectrum, and, thus, the spherical harmonics are not orthogonal. Each mode will contain some of the information from nearby modes. This increase the difficulty of our analysis [217]. Another problem is that the systematic error among different observers has also been found [218,219]. However, the idea of detecting solar oscillation frequencies from the variation of solar wind flux [220] is most likely not to be accepted, because the velocity amplitude they implied is much higher than that detected from solar surface [221].

One of the most exciting results from solar oscillation observation in 1990s is the discovery of the systematic variation of solar p-mode frequencies with solar circle. The frequencies are found to be $(0.46 \pm 0.06)\mu$Hz bigger at solar maximum than minimum [100,170,222–227]. As the $\delta\mu_{\rm nl} = \mu_{\rm nl} - \mu_{\rm n-1,l+2}$ does rarely change, and the change of frequencies for low degree modes is 30% higher than high degree modes [228], such variation are thought to be mode l dependent non-spherical behavior [225,226,229,230]. However, some other persons regard the variation as frequency but not mode dependent, which has a maximum at 4.3mHz, and decrease rapidly at high frequency, but a little bit gently at low frequency[223,231–233]. The cause of this variation is more likely to be related to the change of magnetic field with solar circle. [170,232–236], so it will diminish at solar minimum [237]. While some others suggest the connection of this variation with solar interior structure or the change of size of the cavity - the Sun - itself [222,225,238], or the low degree p-mode modulation [239]. However, the change of solar radius is more likely to be ruled out [132].

Because solar oscillation is thought to be excited by stochastic turbulent motion in the convection zone (see the excitation mechanism section below), solar p-modes have limited life time. The life time is degree n and l dependent, from several hours for high l modes to several months for low l low n modes [208,240]. We can study the correlation of the oscillation signals between two different points on the solar surface, and then use local sound wave property to study the structure between the paths [241,242]. This has become a new way to study the local structure of the Sun.

As the solar g-modes can propagate only in the center of the Sun, its oscillation amplitude is so small in the photosphere that it is hardly observed. The 160 minutes oscillation detected in 1970s is highly doubted as the g-modes, and more high resolution observations are still needed before the real g-modes are observed. Hence in this paper we'll concentrate on the progress of the solar p-modes studies.

3 Progresses in Theoretical Study

3.1 Oscillation nature and boundary condition

To study the theoretical frequencies, usually we need to construct a solar model, which has the same luminosity, radius, mass, age, and surface elements abundance as what we observed, from stellar structure and evolution theory (see paper I). Then, based on this model and the exact boundary condition, we can solve stellar pulsation equations to get the excitation and propagation property of sound waves [243,244]. As the oscillation time scale is much less than the solar evolution nuclear time scale, the model is regarded as not changed during oscillation calculation. At the outer quarter in radius the Sun is in convection instability. The dynamic adjustment time scale at this area is about 1 year, while the cooling time scale is about 10^5 years, so even in the 11 years' solar circle the convection zone can still be treated as in thermal equilibrium. The convection zone can absorb or emit heat to adapt the luminosity disturbance without changing its hydrostatic state [245]. So the hydrostatic model is still valid in solar case.

The amplitude of the solar p-mode is so small, which is only 10^{-6} the luminosity of the quiet Sun, that one usually thinks that non-linear effects are negligible. Linear theory is assumed to be a very good approximation [207,217]. Besides the outside layer, adiabatic approximation is satisfied in most of the solar interior. The non-adiabatic effect is, although not negligible, still not a very big factor [207]. Hence sometimes adiabatic frequencies in polytropic model are used as a reference when further improvements are introduced into solar modeling [246].

The use of observational frequencies to inverse solar interior sound speed, density and rotation velocity along radius and latitude direction shows some very interesting results [247], while the theoretical frequencies have still several μHz difference with observations, which is much bigger than the observation error. The theoretical frequencies are a little bit smaller for low degree modes, while bigger for high degree modes. Such discrepancy is thought to be due to the ignorance of non-adiabatic effect, dynamic effect in convection zone, excitation mechanics, solar atmosphere behavior, and magnetic fields in oscillation study [5,7,14]. As most of these influence located in the surface layer, more details on the behavior of sound wave near solar surface will be needed. However, for those low degree modes and $\delta\nu_{nl}$, the behavior in solar interior can also not be neglected [172].

As we know, the contribution of solar material on sound wave in one area depends mainly on the time the wave costs in that region. The longer time the wave stays, the more influence it will get. As the sound speed in solar surface area is much smaller than it is in the solar center, p-mode will use longer time in the surface area than in the center. So the surface non-adiabatic effect may have obvious influence [7,169]. The modulation of solar luminosity perturbation [231] and g-mode [248] on solar p-mode is somehow also needed to be considered.

The outer reflection point for p-modes locates in between solar photosphere and chromosphere, so the contribution of chromosphere is important, especially for those high degree modes with frequencies near the 5.3mHz cut-off frequency [28,138,206,249–252]. The reason that solar atmosphere connection condition does not have significant influence on low degree modes [45] is the cancellation of several different mechanisms [253]. While, the introduction of chromosphere magnetic field in frequency variation with solar circle seems to have some problem [254,255]. The validity of diffusion approximation and Eddington approximation in out layer [121,256,257], and the reflection and transmission of surface sound wave [258] need further considerations, too.

3.2 Theoretical frequencies

By calculating solar oscillations frequencies, we can distinguish the validity of those important theories in solar modeling, such as high temperature plasma physics, magnetohydrodynamics, neutrino theory, radiative transfer, convection and rotation [230]. Although the observational and theoretical frequencies can fit in 0.1%, the discrepancy is obviously systematic. How to overcome such a difference is one of the most important goals for theoretical study. In this section we'll focus on how the improvement of input physics affects this discrepancy.

The input physics in solar modeling may modify the structure of the Sun, and, thus, change the eigenvalue. Among them, the uncertainty in the equation of state plays a very important role [216,259]. As the convection contains less influence from the uncertainty of other input physics, the behavior of frequencies in convection zone for different equations of state becomes the most important factor in distinguishing different equations of states [260]. It is found that, once the Debye-Hückel effect is considered in equation of state, the observation–calculation (O–C) value will significantly decrease [17–19]. The result for the MHD equation of state is similar to that of Debye-Hückel effect included in simple equations of state, while the O–C value becomes flatter, and, thus, better [5,7,17,18,207], while further improvement in the MHD equation of state[28] shows less effect [31]. On the other hand, the improvement of frequencies from the equation of state seems to be limited to those $40 \leq l \leq 200$ intermediate degree modes, and will be worse for those higher or lower degree modes [30]. The difference in frequencies for the generally very closely lying MHD and OPAL equations of state has also been found [34], and the later is sometimes favored.

The improvement of opacity has also some contribution. The LAAOL opacity are found to be too small at the base of convection zone [7,174,175,207]. The OPAL opacity, which considers the contribution of iron, gets an enhancement at this area, and, hence, improve the intermediate degree frequencies [18,261], while for low degree modes the effect is too small. It seems that the opacity itself does not solve the O-C difference [18,29,261]. If the opacity at the bottom of the convection zone can get further increase, and the low temperature opacity below 9000K can also be increased, then we can get much more improvements [7,18,26,31,205], while the physical reasons for such enhancements are still to be discovered.

As the solar surface helium abundance from inversion is lower than that of standard model, the diffusion process for helium and heavy elements has been reconsidered in 1990s. The most significant result for this process is in the helium second ionization zone, where those $l = 300 - 600$ intermediate degree modes are sensitive to. The existence of diffusion are thought to improve the theoretical frequencies [22,107,109,118,164], while sometimes the effect is found to be selective, which can only benefit those convective base sensitive modes but will deteriorate other modes [21,92,106,113,121,175]. The diffusion is also discouraged by some inversion studies [95].

For other factors, the pre-main sequence evolution is found to have less influence on frequencies [28], and the age uncertainty is still a small factor for frequencies when comparing with other input physics [28,129], while if the solar age can be as old as 5.2Gyr, then the frequencies will be obviously influenced [18,109]. The possibility of changing gravity constant with time is small [130,131]. The global magnetic field, while, will have very small effect to the frequencies [72].

The arbitrary redistribution of hydrogen, helium or heavy element in solar interior may sometimes improve some modes [138,205,262], while, when the whole solar model and all the frequencies are taken into account, such an arbitrary redistribution of elements, and the introduction of WIMPs into solar core to reduce the solar neutrino, which has similar result, are somewhat hard to accept [7,21,39,170,171,174,175].

When strong solar wind in pre- or early main sequence stage is introduced to analyse the solar surface lithium depletion, from the point of view of solar oscillation, such a mass loss must eject about $0.1M_\odot$ mass in less than 0.2Gyr [36,119,149].

3.3 Inversion

From observation we can now get as many as 10^7 different oscillation modes. We can use the ordinary stellar oscillation theory to calculate the theoretical frequencies, and then compare them with the observed value, just as in last section. But we can also relax some basic assumptions in the stellar oscillation theory, then use those observed frequencies to construct a cavity which can allow these waves to travel as standing waves. This is the so-called seismic inversion method. The obtained solar seismic model can then be compared with theoretical model to study the difference. In order to do the inversion, we could use observed frequencies, amplitude and phase information, eigenfunctions, or the influence to eigenfunctions during its propagation. Presently, only frequencies are popularly used [117].

There are basically two different methods in solar oscillation inversion. One is the so-called asymptotic inversion. If the solar structure is a function of sound speed and the dispersion of the sound waves, i.e. ω and l, then, by using this relation:

$$F(w) + \frac{1}{\omega^2}P(w) = \frac{\pi[n+\alpha(\omega)]}{\omega},$$

$$w = \omega/(l+\frac{1}{2}),$$

in which $F(w)$ is a function of sound speed, $P(w)$ includes all the second terms, $\alpha(\omega)$ represent the surface phase shift, we can get the sound speed distribution inside the Sun. Then we can plug in the known input physics, i.e. equation of state, opacity, and nuclear reaction rates, to get the solar structure of the inverse area [35,263–272]. While, whether the gravity potential perturbation Φ' should be included or not (Cowling approximation) is still a question in this method [169,273].

Another method is the direct numerical method. Once we can get a theoretical solar model, we can then find out the difference in density, sound speed, adiabatic exponent and chemical abundance between our conceived model and the theoretical model from the observed and theoretical frequency difference (O–C),

$$\frac{\delta\omega_{nl}}{\omega_{nl}} = \int_0^R [K_{nl}^{c,\rho}(r)\frac{\delta c(r)}{c} + K_{nl}^{\rho,c}(r)\frac{\delta\rho(r)}{\rho}]\mathrm{d}r + E_{nl}^{-1}G(\omega_{nl}) + \epsilon_{nl}$$

where $G(\omega_{nl})$ is the surface effect [35,105,270,274–276]. There are some different techniques to do this inversion, such as RLS (regular least square), OLA (optimal localized average), or SOLA (subtractive optimal localized average) [79,274,277,278].

One of the most difficult problems in inversion is how to treat the surface reflection phase shift caused by non-adiabatic convection at the near surface area of the Sun. This effect, although not very big, is found to be not negligible [105,279–281]. So it is sometimes represented by a slowly changed frequency dependent function [274,282–284]. While it is found that this effect is not just frequency dependent in higher accuracy [285]. Hence a more effective way is to filter this surface effect out before inversion [286–288]. As there is a resolution limit for the fixed number of modes, and the low degree modes that can travel through the solar core are difficult to observe, the inversion result will have less accuracy when it is applied onto solar center area [79,143]. More different frequency modes from observation are demanded to increase the resolution [289,290].

From inversion it is found that the change of temperature gradient in the helium second ionization layer in the solar convection zone may have significant influence on sound speed, and this area is also sensitive to the equation of state, so the inversion result can be used to examine the validity of different equations of state [144,291–295]. The result supports the adoption of the Debye-Hückel correction or the use of the MHD equation of state [296], while the OPAL equation of state is sometimes more favored [297].

The inversion results approved the modification in OPAL opacity [289,298], while, as the inversed solar center temperature and neutrino fluxes are higher than observation [163,295,299], the opacity in the solar center is suggested to be reduced [68] (See paper I section 8 for related discussion). The inversion results support the existence of helium and heavy elements diffusion process [100,297,300–302], while the change of the convection zone structure should not be ignored [303]. The age difference with the standard model is another problem [295,304].

The most serious problem in inversion is that, because the frequencies are not so sensitive to the change of solar structure, the very small systematic error in observation may cause a significant change in inversed structure. The differences in solar center structure have been found from inversion results with the observation data from different groups [143,302,305]. The believability of the inversion structure, especially to the solar center area, is somewhat questionable before more high accuracy and consistent observational data are available.

3.4 Excitation mechanism

The excitation mechanism in solar oscillation refers to the physical processes that transfer energy from other forms into the pulsational mechanic energy. It is found that the normal κ mechanism, which is effective in other variable stars can not be used into the solar case, because the radiation and convection processes will damp the oscillation in the Sun. The Sun is, thus, not self-excited. The κ mechanism also can not explain the observed f-mode and the small amplitude. Because the amplitude is so small that the non-linear effect can not develop to restrain it, and, hence, the predicted amplitude should be much bigger than what we get from observation [270,306–310].

It is popularly believed that these millions of solar p-modes are resonance eigenfunctions excited by the random motion of the convective turbulence [230,240,247,258,311–314], while the g-modes are thought to be excited by turbulent stresses [221]. The stochastically excited and intrinsically damped oscillator model can produce the observed power spectrum, too, even though the observed spectra profile has not been fitted yet [35]. Because of the turbulence viscosity dispersion, the leakage of energy at the reflecting boundary, the transformation of energy among modes by non-linear effects, and the non-adiabatic effect, the oscillation will be damped. Among these effects, the radiative flux gradient dominates the damping process [208,311]. The amplitude and the oscillation spectrum peak equivalent width can be explained by the efficiency of the excitation and damping processes [207,306]. Because the modes are stochastically excited, their life time is limited. The p-mode life time is from several hours for high frequency modes to several months for low frequency modes [208,240].

Another interesting topic in excitation mechanism is the location of excitation source. By comparing the appearance of spectrum intensity with asymmetry profile for different degree modes, the location of excitation source can be inversed out [315,316]. Usually it is thought that the source is at the turbulent subsurface convective boundary layer [116,311,317], and, furthermore, it is just at the cooling upper layer but not at the overshooting zone [318]. As the asymmetry of the p-mode spectrum depends on the location of the excitation source, and frequency, and weakly on degree, but not on the excitation mechanism and the location of the damping layer, we can get the dependence of the source location on frequencies. It is found that the excitation of 2 to 3mHz low degree modes is at 325km beneath the photosphere, which is much deeper than that of the 6mHz high degree modes [319]. For those frequencies of 5.3mHz, which is near the surface reflection limit, the excitation source is usually found to be 100 – 200km beneath the photosphere, where the convection speed get its maximum [320–323].

3.5 Rotation

From observation we have known for a long time that the solar surface is in differential rotation, where the rotation speed is different in different depth and latitude. The rotation speed distribution in solar deep interior is thus a very interesting topic, and the existence of solar p-modes gives us an opportunity to directly inverse the rotation profile of the Sun.

Because of rotation, the degeneracy of the solar p-modes is released. Those that have the same n and l order but different m degree modes will have a little difference in their frequencies. By studying this difference with frequency and degree, we can then inverse the internal rotation profile along the radial and latitude direction [207,230,289,324].

The frequency splitting caused by rotation is about 462nHz at $r = 0.8R_\odot$ area, and will increase to 475nHz at $r = 0.92R_\odot$, then decrease to 452nHz at the surface [325]. We know that

$$\triangle\nu_{nlm} = -\frac{m}{2\pi}\int\int K_{nlm}(r,x)\Omega(r,x)\mathrm{d}r\mathrm{d}x,$$

where $\triangle\nu$ is the influence of that wave by its propagation area. So we can get the radial and latitude distribution of the solar rotation [326]. The result, while, is not so accurate for high latitude ($> 60^o$) area [326,327]. Because the signal to noise ratio for low degree mode frequency split is small, and the contribution for such split from solar center is small, too, the resolution for solar center rotation is limited [328]. Besides the rotation, the size and life time of the solar spots may also influence the frequency split [326].

From rotation inversion we now know that the solar surface differential rotation extends to the whole convection zone. At the base of the convection zone of $r \sim 0.7R_\odot$ there is a discontinuity in rotation. Beneath that the Sun rotates as a solid body, without any dependence on latitude [326,328–330]. However, some work suggests that the center solid body rotates a little faster than the surface equator [170,326,331], while some others think that the speed is a little lower than the surface equatorial speed [329,332].

3.6 Convection zone

From theoretical model the solar convection zone locates outside the $r = 0.708$ $R_\odot$ area [118], while from the inversion of sound speed the convection base is suggested to be at $r = (0.713 \pm 0.003)R_\odot$ [113,262,281,292,300]. It seems that they can almost coincide with each other, although much deeper result of $r = 0.68R_\odot$ from inversion is also obtained [333].

It is found that theoretical frequencies will rarely be influenced by the change of convective efficiency α or turbulent pressure in standard mixing length theory (MLT) [22,28]. While, if the convective efficiency is decreased in the model, the theoretical frequencies may increase [334], and the increase of efficiency will improve the theoretical frequencies [335]. Sometimes the increase of the temperature gradient in the convection zone can even improve the theoretical frequencies [141]. The use of a temperature dependent $\alpha(T)$ MLT also seems to be useless [46]. When turbulence spectrum in local convection theory, which describes the convection boundary more accurately, is considered, the theoretical and observational difference is reduced, while the difference is still much bigger than observation error [80,81,83,303,336].

As almost a quarter of the solar outside radius is convection, and all the p-modes must travel through this area, especially for those high degree modes, which will mostly propagate in this zone, how to treat the convective motion with the oscillation will be extremely important [201,245,335,337]. At the most outer layer of the convection zone, the non-adiabatic effect is so strong that the adiabatic approximation is invalid. The non-adiabatic effect on the boundary condition, and, thus, on the frequencies, should be carefully studied [7,305,339–341].

If there is overshooting layer at the inner boundary of the convection zone, the mixing length will increase, and, thus, the pressure and density distribution of the convection zone will change. So the frequencies will change, too, and the effect depends on the size of the overshooting layer[207]. The existence of overshooting may increase all the frequencies [89,90], and may improve some of the theoretical modes [93]. This effect will be enhanced if the non-sphere effect on the Sun is considered [325]. While the $0.2H_\mathrm{p}$ overshooting seems not enough to solve the frequency problem [31]. If there is overshooting layer, the second derivative of the sound speed will be discontinuous. The sound wave will then have extra phase shift. Some inversion result supports the $0.2H_\mathrm{p}$ overshooting zone [94,342,343], but others are against such possibility [86,87,344,345], or regard it as less than $0.07H_\mathrm{p}$ [95,346,347].

3.7 Sound speed and helium abundance

The sound speed is one of the rare quantities that can be inversed directly without using other input physics, and its inversion result can be as accurate as 0.1% [116,264]. The sound wave may have some phase shift when it travels through density discontinuity region [348]. We can then use this property to detect the depth of the solar convection and overshooting zone [95,170], and to distinguish those different input physics in solar modeling [6,7,263,303,304,325]. From the sound speed inversion, we have even discovered the systematic discrepancy between different observation frequencies [143,218].

The solar surface helium abundance is one of the very important but uncertain things. As the solar helium second ionization zone is still inside the convection zone, we can use the frequencies to inverse the adiabatic exponent here, then use the equation of state to get the helium abundance in the convection zone [22,35,100,293]. The helium abundance is different when different equation of state is used. From the simple equation of state, the result is $Y = 0.254 - 0.257$, from the MHD equation of state it is $Y = 0.23 - 0.25$ (The big error comes from the different methods used by different authors) while from the OPAL equation of state it is $Y = 0.25$ (The result is so exact because there's only one author who uses this equation of state to do such inversion). All of these values are smaller than the predicted value in theoretical model, which is $Y = 0.28$[23,29,35,100,105,113,118,143,144,283,284,291,325]. The inversion result from intermediate degree modes is even smaller than that from the low degree modes [333]. All these support the introduction of helium and heavy elements diffusion process into solar modeling. The effect of diffusion is discussed in paper I and the above section 3.2.

4 Conclusion and Discussion

By studying solar seismology, we have known much more than before about the internal structure of the Sun. This is a fundamental test of the theory of stellar evolution, one of the most important astrophysical tools to study the universe. The positive result from oscillation shows that our standard model exhibits no gross mistakes. The helioseismic studies even help us to rule out some unreasonable or not existing assumptions and processes which are introduced to explain some exact phenomena, such as the arbitrary enhancement of the hydrogen abundance in the solar core. The frequencies are now one of the most powerful discriminators among different models, and even a testing stone in distinguishing those input physics, such as equations of state and opacity, which are sometimes impossible to check in ground laboratories.

Beyond the success, we need to notice that there is still much work to do, especially how to explain the discrepancy between observational and theoretical frequencies. Even when a lot of processes have been taken into account, and sometimes some of them are introduced artificially, the best fitted results from theoretical models are still different from those observational data. How to improve the accuracy of our models reasonably is still a big question (See paper I). Because sometimes the artificial increase of the weight of some actually not so important factors may, accidentally, improve some of our results. The theory, but not the best fit, can be accepted only if we can prove it from physics theory and the actual condition in the Sun.

Besides the study of frequencies from solar model, we can also inverse the model from the observed frequencies. The inversion method is valid to all those area that pulsation waves travel through. The more modes to propagate through an area, the more accurate the inversion result for that region will be. That is why at this time the inversion result for solar center is still less accurate, because there are less low degree modes which can travel to the solar core, and their signal to noise ratio from observation is still low. How to prevent those observation data which have big error to influence our inversed result also needs to be considered.

The effect of those discontinuous and non-adiabatic layers on the inversion result is also a key point on whether the inversed model is accurate or not. As the adiabatic approximation is not valid at the most outer layer of the convection zone, the non-adiabatic effect can not be neglected in either theoretical oscillation study or inversion model. The problem is that there are so many possible ways to solve the non-adiabatic effect. In order to solve the non-adiabatic equations, some simplification is unavoidable. How to consider such effect in our analyses becomes one of the most interesting things now.

If we just want to filter out the surface non-adiabatic effect before inversion, then we need to make sure which result comes from non-adiabatic effect, and which does not. For those high frequency modes, the possibility of energy leakage from photosphere and the contribution of chromosphere with p-modes may also influence their behavior.

The variation of frequencies with solar circle, the detail process for the excitation of oscillation, and the location of excitation source are also needed to be studied. Such study can let us know more nature about the energy transfer in the Sun. While, one of the most challenging problems is how to consider the rotation and magnetic field in our global and local solar structure study, which will need a lot of complicated analyses.

From observation, in addition to the improvement of accuracy, we are waiting for more new modes to be detected, especially in those high and low degree modes. The systematic discrepancy among different group's data should also be considered, so that we can get consistent result. Because a little difference in frequencies may cause a large difference in our model. Another challenging topic is to detect the g-modes, which can then give us more information on solar core.

One of the most important goals of helioseismology is to use the knowledge we get from the Sun to understand other stars. As we mentioned above, it has become a formidable test of the theory of stellar evolution. It will also be an important benchmark for astro-seismology, the emerging field of multiperiod pulsating stars. The more accurate our knowledge about the Sun, the better we can understand other stars. And other stars will teach us about the past and future of our Sun. The whole process will enhance our understanding of the universe and its evolution.

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